Name: ______________________

Teacher: ______________________

Class: ______________________

FORT STREET HIGH SCHOOL

**2022** **HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION**

# Mathematics Extension 1

**General Instructions**

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- NESA-approved calculators may be used
- A reference sheet is provided at the back of this paper.
- For questions in Section II, show all relevant reasoning and/or calculations.

**Total marks – 70**

- **Section I -10 marks**
- Attempt Q1-10
  Allow about 15 minutes.

- **Section II-60 marks**
  Attempt Q11-16
  Allow about 1 hour 45 minutes.

## Section I

**10 marks**
**Attempt Questions 1–10**
**Allow about 15 minutes for this section**

Use the multiple-choice answer sheet for Questions 1–10.

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**1.** A curve is described by the parametric equations below.

$$x = \frac{t}{2} \qquad y = 3t^2$$

What is the cartesian equation of the curve?

A. $x = 12y^2$

B. $y = \frac{3x^2}{4}$

C. $y = \frac{2}{x}$

D. $y = 12x^2$

**2.** Which set of values for $a$ and $b$ satisfy the following equation?

$$\begin{pmatrix} 3a - 4b \\ 2a + 3b \end{pmatrix} = \begin{pmatrix} -16 \\ 12 \end{pmatrix}$$

A. $a = 0$ and $b = 4$

B. $a = 1$ and $b = \frac{19}{4}$

C. $a = \frac{9}{2}$ and $b = 1$

D. $a = 4$ and $b = 0$

**3.** Jack starts at the origin and walks along vector $2\underset{\sim}{i} + 3\underset{\sim}{j}$ and then turns and walks along vector $4\underset{\sim}{i} - 2\underset{\sim}{j}$. How far is Jack from the origin ?

A. 5

B. $\sqrt{11}$

C. $\sqrt{37}$

D. $\sqrt{61}$

**4**. What is the domain and range of the function $y = 6\,sin^{-1}(2x)$ ?

A. Domain $\left[-\frac{1}{2}, \frac{1}{2}\right]$ ; Range $[-3\pi , 3\pi]$.

B. Domain $\left[-\frac{1}{2}, \frac{1}{2}\right]$ ; Range $[-6\pi , 6\pi]$.

C. Domain $[-6\pi , 6\pi]$ ; Range $\left[-\frac{1}{2}, \frac{1}{2}\right]$.

D. Domain $[-3\pi, 3\pi]$ ; Range $\left[-\frac{1}{2}, \frac{1}{2}\right]$.

**5.** A library contains 10 different maths books and 15 different science books.
In how many ways can a group of three maths and two science books be borrowed?

A. 250

B. 900

C. 12 600

D. 20 475

**6.** What is the derivative of $\cos^{-1}(x^3)$ with respect to $x$?

A. $\dfrac{1}{\sqrt{1-x^6}}$

B. $-\dfrac{3x^2}{\sqrt{1-x^6}}$

C. $-\dfrac{1}{\sqrt{1-x^6}}$

D. $\dfrac{3x^2}{\sqrt{1-x^6}}$

7. What is the value of $\sin 2x$ given that $\sin x = -\dfrac{4}{5}$ and $x$ is an angle in the third quadrant?

A. $-\dfrac{12}{25}$

B. $-\dfrac{24}{25}$

C. $\dfrac{12}{25}$

D. $\dfrac{24}{25}$

**8.** If the roots of $P(x) = x^2 + 5x + k + 1$ are consecutive integers then the value of $k$ is

A. -7

B. -5

C. 5

D. 7

**9.** A particle currently has positive displacement and negative velocity and always has constant positive acceleration. Which one of the following statements must be true about the particle?

A. Its speed is currently increasing.

B. Its speed is currently decreasing.

C. Its velocity will always be negative.

D. Its displacement will eventually become negative.

**10.** $g(x)$ is the inverse function of $f(x) = e^{x-1}$. Which one of these statements must be true for all $x$ in the domain of $g(x)$ ?

A. $g(x) > 0$

B. $g(x) < 0$

C. $g''(x) < 0$

D. $g'(x) < 0$

**End of Section I, Multiple Choice,**
**Section II begins on next page.**

## Section II

**60 marks**
**Attempt Questions 11–16**
**Allow about 1 hour and 45 minutes for this section**

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

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**Marks**

**Question 11** (10 marks) Use a SEPARATE writing booklet

(a) Find the angle between the vectors $\underset{\sim}{u} = {}^{-}2\underset{\sim}{i} + 6\underset{\sim}{j}$ and $\underset{\sim}{v} = 4\underset{\sim}{i} - 2\underset{\sim}{j}$ **2**

(b) Find the equation of the curve $y = f(x)$ that has $f\,'(x) = \dfrac{1}{\sqrt{9 - x^2}}$ and passes through $(3,0)$ . **2**

(c) When the polynomial $P(x)$ is divided by $4x^2 - 9$ the remainder is $8x - 5$ . What is the remainder when $P(x)$ is divided by $2x - 3$ ? **2**

(d) i) Write $3\cos x - \sqrt{3}\,\sin x$ in the form $R\cos(x + \alpha)$ **2**

ii) Hence solve $3\cos x - \sqrt{3}\,\sin x = \sqrt{3}$ for $0 \le x \le 2\pi$ **2**

**End of Question 11**

**Question 12 (10 marks) Use a SEPARATE writing booklet** **Marks**

(a) Use $t$-identities to solve $\sqrt{3}\sin\theta - \cos\theta = 1$ for $[0, 2\pi]$ **4**

(b) Use the substitution $u = \sin x$ to find $\int -\cos x \sin^{\frac{3}{2}} x \, dx$ **2**

(c) Prove by mathematical induction that $2n(n-1)$ is divisible by 4 for all positive integers $n > 1$ . **4**

**End of Question 12**

**Question 13 (10 marks) Use a SEPARATE writing booklet** Marks

(a) A particle, experiencing vertical acceleration due to gravity $g$ and no air resistance, is projected over horizontal ground at speed $v\ ms^{-1}$ at an acute angle $\theta$ to the horizontal. You may assume the six equations of motion listed below.

$$\ddot{y} = -g\underset{\sim}{j} \qquad \ddot{x} = 0$$

$$\dot{y} = (-gt + v\sin\theta)\underset{\sim}{j} \qquad \dot{x} = v\cos\theta\,\underset{\sim}{i}$$

$$y = \left(-\frac{g}{2}t^2 + vt\sin\theta\right)\underset{\sim}{j} \qquad x = vt\cos\theta\,\underset{\sim}{i}$$

i) Show that the

α) time to reach greatest height is $\dfrac{v\sin\theta}{g}$ **1**

β) greatest height is $H = \dfrac{v^2\sin^2\theta}{2g}$ **1**

γ) time to reach the landing point is $\dfrac{2v\sin\theta}{g}$ **1**

δ) range is $R = \dfrac{v^2\sin 2\theta}{g}$ **1**

ii) If $R$ is three times $H$, find the exact angle of projection. **2**

**Question 13 continues on the next page**

**Question 13 (continued)**

Marks

(b) The rate of change in temperature, T degrees celsius, of a metal over time, t minutes, as it cools is given by the equation $\frac{dT}{dt} = -k(T - 25)$ . The metal is initially at $300°C$ and cools to $250°C$ after 5 minutes.

i) Show that $T = 25 + Ae^{-kt}$ is a solution of the equation $\frac{dT}{dt} = -k(T - 25)$ **1**

ii) Find the values of $A$ and $k$. **1**

iii) After how many minutes will the temperature of the metal be $100°C$ ? Give your answer to the nearest minute. **2**

**End of Question 13.**

**Question 14 (10 marks) Use a SEPARATE writing booklet** **Marks**

(a) The volume $V$ and surface area $S$ of a sphere of radius $r$ are given by

$V = \frac{4}{3}\pi r^3$ and $S = 4\pi r^2$

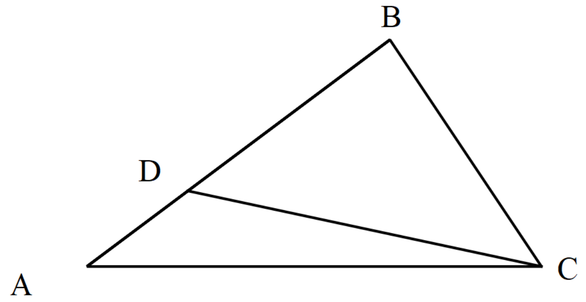
i) Show that $\frac{dV}{dr} = S$ and $\frac{dV}{dt} = S\frac{dr}{dt}$. **2**

ii) A spherical ice ball of radius 24mm is immersed in water. The time it takes for its volume to decrease is measured in minutes. Its volume decreases at a rate equal to three times its surface area. If the ice ball is always spherical, how much time does it take to reduce its radius to one-eighth its original radius? **2**

**2**

(b) Find the vector projection of $\underset{\sim}{u} = 3\underset{\sim}{i} + 2\underset{\sim}{j}$ onto $\underset{\sim}{v} = -\underset{\sim}{i} + 2\underset{\sim}{j}$

(c) In $\Delta ABC$, $D$ is a point on $AB$. where $|\overrightarrow{AD}| : |\overrightarrow{DB}| = 2 : 3$




Given $\overrightarrow{AD} = \underset{\sim}{a}$, $\overrightarrow{AC} = \underset{\sim}{b}$ and $\overrightarrow{CB} = \underset{\sim}{c}$, show that $\underset{\sim}{b} = \frac{1}{2}(5\underset{\sim}{a} - 2\underset{\sim}{c})$ **2**

(d) A father, a mother and five children stand in a circle. In how many ways may they be arranged so that the father and mother do not stand together. **2**

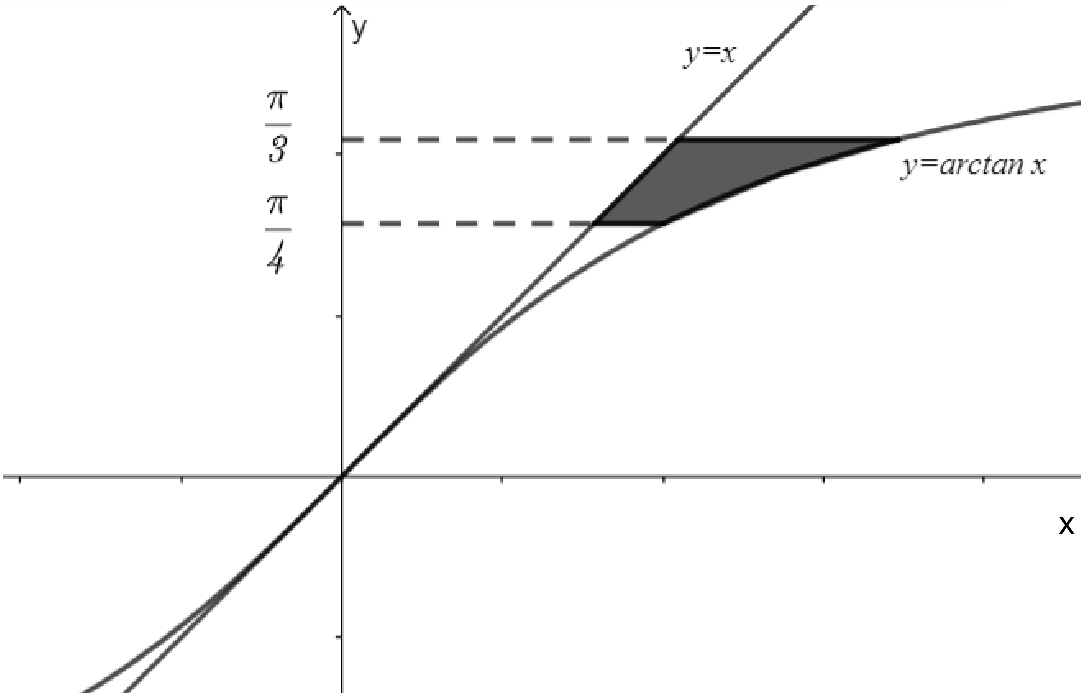
**End of Question 14**

**Question 15 (10 marks) Use a SEPARATE writing booklet** **Marks**

(a) Find $\int \cos^2 \frac{x}{3}\, dx$ **2**

(b) Find the volume formed when the region between the curve $y = \tan^{-1} x$ **4**

and the lines $y = x$ , $y = \frac{\pi}{3}$ and $y = \frac{\pi}{4}$ , is rotated around the $y$-axis.



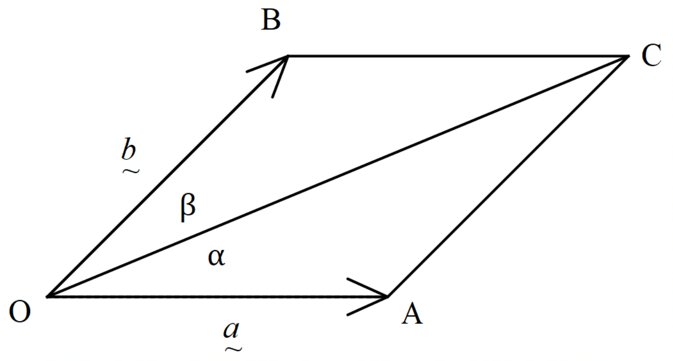

(c) i) Show that $\frac{d}{dx}\left[x\sin^{-1}x + \sqrt{1-x^2}\right] = \sin^{-1}x$ **2**

ii) Hence, show that $\int_{\frac{1}{2}}^{1} \sin^{-1}x\, dx = \frac{5\pi - 6\sqrt{3}}{12}$ **2**

**End of Question 15**

**Question 16 (10 marks) Use a SEPARATE writing booklet** **Marks**

(a) $OACB$ is a rhombus. $\overrightarrow{OA} = \underset{\sim}{a}$ , $\overrightarrow{OB} = \underset{\sim}{b}$ , $\angle AOC = \alpha$ and $\angle COB = \beta$




i) For the vectors in the diagram above, use properties of the dot product to prove $\underset{\sim}{a} \cdot (\underset{\sim}{a} + \underset{\sim}{b}) = \underset{\sim}{b} \cdot (\underset{\sim}{a} + \underset{\sim}{b})$ **2**

ii) Hence, prove diagonal $OC$ bisects $\angle AOB$ **2**

(b) i) Prove the identity $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ **2**

ii) Hence, find exact value expressions for three unique solutions of **4**

$$8x^3 - 6x = -\sqrt{3}$$

**End of paper**

**2022 FSHS Trial HSC Examination**
**Mathematics Extension 1 Course**

Name SOLUTIONS Teacher ____________

## Section I – Multiple Choice Answer Sheet

**Allow about 15 minutes for this section**

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

| | | | | |
|---|---|---|---|---|
| 1. | A ○ | B ○ | C ○ | D ● |
| 2. | A ● | B ○ | C ○ | D ○ |
| 3. | A ○ | B ○ | C ● | D ○ |
| 4. | A ● | B ○ | C ○ | D |
| 5. | A ○ | B ○ | C ● | D ○ |
| 6. | A ○ | B ● | C ○ | D ○ |
| 7. | A ○ | B ○ | C ○ | D ● |
| 8. | A ○ | B ○ | C ● | D ○ |
| 9. | A ○ | B ● | C ○ | D ○ |
| 10. | A ○ | B ○ | C ● | D ○ |

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11(a)

$$\cos\theta = \frac{u \cdot v}{|u| \times |v|}$$

$$= \frac{-2\times 4 + 6\times -2}{\sqrt{(-2)^2+6^2}\times\sqrt{4^2+(-2)^2}}$$

$$= \frac{-20}{2\sqrt{10}\times 2\sqrt{5}}$$

$$= \frac{-20}{20\sqrt{2}}$$

$$\cos\theta = \frac{-1}{\sqrt{2}}$$

| | |
|---|---|
| ✓S | A |
| ✓T | C |

✓

$$\theta = 180-45 \quad \text{or} \quad 180+45$$

$$\theta = 135^\circ \quad \text{or} \quad 225^\circ$$

$\theta = 135^\circ$ as angle between lines $< 180^\circ$

($180 - 135 = 45^\circ$ is also correct)

11 b)

$$f(x) = \int \frac{1}{\sqrt{9-x^2}}\,dx$$

$$= \sin^{-1}(x/3) + c$$ ✓

When $x = 3$, $y = 0$

$$\therefore \quad 0 = \sin^{-1}(1) + c$$

$$= \pi/2 + c$$

$$c = -\pi/2$$ ✓

$$y = \sin^{-1}(x/3) - \pi/2$$

11 c) $P(x) = (4x^2-9)\,Q(x) + 8x-5$ ✓

$= (2x+3)(2x-3)\,Q(x) + 8x-5$

$P\left(\frac{3}{2}\right) = 0 + 8\left(\frac{3}{2}\right)\ 5$

$= 7$

$\therefore$ Remainder, when $P(x)$ is divided by $2x-3$, is $7$ ✓

11 d) i) $3\cos x - \sqrt{3}\sin x \equiv R\cos(x+\alpha)$

$= R\cos x\cos\alpha - R\sin x\sin\alpha$

Equating co-efficients of like terms

$3 = R\cos\alpha \ldots(1)$ and $\sqrt{3} = R\sin\alpha \ldots(2)$

$(1)^2 + (2)^2$ $\quad 12 = R^2(\cos^2\alpha + \sin^2\alpha)$

$2\sqrt{3} = R$ (assuming $R>0$) ✓

$(2) \div (1)$ $\quad \tan\alpha = \frac{1}{\sqrt{3}}$

$\alpha = \pi/6$ (assuming $0<\alpha<\pi/2$) ✓

$\therefore\ 3\cos x - \sqrt{3}\sin x \equiv 2\sqrt{3}\cos(x+\pi/6)$

11 d) ii) $3\cos x - \sqrt{3}\sin x = \sqrt{3}$

$2\sqrt{3}\cos(x+\pi/6) = \sqrt{3}$

$\cos(x+\pi/6) = \frac{1}{2}$ ✓

$x+\pi/6 = \pi/3$ or $5\pi/3$

$x = \frac{\pi}{6}$ or $\frac{3\pi}{2}$ ✓

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12 a) $\sqrt{3}\sin\theta - \cos\theta = 1$

$\sqrt{3}\,\dfrac{2t}{1+t^2} - \dfrac{(1-t^2)}{1+t^2} = 1$ ✓

$2\sqrt{3}t - 1 + t^2 = 1 + t^2$

$2\sqrt{3}t = 2$

$t = 1/\sqrt{3}$ ✓

$\tan\theta/2 = 1/\sqrt{3}$

$\theta/2 = \pi/6$ or $\pi + \pi/6$

$\theta = \pi/3$ or $7\pi/3$

but $0 \le \theta \le 2\pi$

$\therefore \theta = \pi/3$ ✓

Also check $x = \pi$,

LHS $= \sqrt{3}\sin\pi - \cos\pi$

$= 0 - -1$

$= 1$

$=$ RHS

$\therefore$ Solution is $x = \pi/3$ or $\pi$. ✓

12 b) $u = \sin x$

$du = \cos x \, dx$

$\therefore \int -\cos x \sin^{3/2} x \, dx$

$= - \int u^{3/2} \, du$

$= - \dfrac{u^{5/2}}{5/2} + c$

$= -\frac{2}{5} \sin^{5/2} x + c$

12 c) For $n=2$, $2n(n-1)$

$= 4(1)$

$= 4$

which is divisible by 4

so true for $n=2$

Assume true for $n=k$

That is, assume $2k(k-1) = 4M$, $M$ an integer

For $n=k+1$, it is required to show that

$2(k+1)(k+1-1) = 4P$, $P$ an integer

That is, $2k(k+1) = 4P$

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12(c) (continued)

$$LHS = 2k(k+1)$$
$$= 2k(k-1+2)$$
$$= 2k(k-1) + 4k$$ ✓
$$= 4M + 4k \quad \text{(by assumption)}$$ ✓
$$= 4(M+k)$$
$$= 4P, \text{ where } P = M+k \text{ is an integer.}$$
$$= RHS$$

So the result is true for $n=k+1$ if true for $n=k$. Hence the result is proven true by mathematical induction. ✓

Marking guideline

1 mark for $n=2$ and stating $n=k$

2 marks for $n=k+1$ step

1 mark for correct conclusion.

13 a) i) $\alpha$) Greatest height occurs when

$$0 = \dot{y}$$

$$0 = -gt + v\sin\theta$$

$$gt = v\sin\theta$$

$$t = \frac{v\sin\theta}{g}$$

i) $\beta$) When $t = \frac{v\sin\theta}{g}$, $y = H$

$$H = -\frac{g}{2}\left(\frac{v\sin\theta}{g}\right)^2 + v\left(\frac{v\sin\theta}{g}\right)\sin\theta$$

$$= -\frac{v^2\sin^2\theta}{2g} + \frac{v^2\sin^2\theta}{g}$$

$$= \frac{v^2\sin^2\theta}{2g}$$

i) $\gamma$) When $y = 0$,

$$0 = -\frac{gt^2}{2} + vt\sin\theta$$

$$0 = t\left(-\frac{gt}{2} + v\sin\theta\right)$$

$$t = 0 \text{ or } \frac{2v\sin\theta}{g}$$

$\therefore$ time to landing point is $\frac{2v\sin\theta}{g}$

i) $\delta$) when $t = \frac{2v\sin\theta}{g}$, Range $= x$

$$\therefore \text{Range} = v\left(\frac{2v\sin\theta}{g}\right)\cos\theta$$

$$= \frac{v\sin 2\theta}{g}$$

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13 a ii) $R = 3H$

$$\frac{v^2 \sin 2\theta}{g} = \frac{3v^2 \sin^2\theta}{2g}$$

$$\sin 2\theta = \frac{3}{2}\sin^2\theta$$ ✓

$$2\sin\theta\cos\theta = \frac{3}{2}\sin^2\theta$$

$$4\sin\theta\cos\theta - 3\sin^2\theta = 0$$

$$\sin\theta(4\cos\theta - 3\sin\theta) = 0$$

$$\sin\theta = 0 \quad \text{or} \quad 4\cos\theta = 3\sin\theta$$

$$\theta = 0 \qquad \frac{4}{3} = \tan\theta$$

$$\theta = \tan^{-1}\left(\frac{4}{3}\right)$$

$\therefore$ Exact angle of projection is $\tan^{-1}(4/3)$ ✓

13b) i) $LHS = \frac{dT}{dt}$

$= \frac{d}{dt}\left(25 + Ae^{-kt}\right)$

$= -kAe^{-kt}$

$RHS = -k(T-25)$

$= -k(25 + Ae^{-kt} - 25)$

$= -kAe^{-kt}$

$= LHS$

$\therefore$ $T = 25 + Ae^{-kt}$ is a solution

b ii) When $T = 300$, $t = 0$

$\therefore 300 = 25 + Ae^{0}$

$275 = A$

When $T = 250$, $t = 5$

$\therefore$ $250 = 25 + 275e^{-5k}$

$\frac{225}{275} = e^{-5k}$

$\frac{275}{225} = e^{5k}$

$\frac{11}{9} = e^{5k}$

$\ln\left(\frac{11}{9}\right) = 5k$

$k = \frac{1}{5}\ln(11/9) \doteqdot \cdot 040134$

✓

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13b iii) $100 = 25 + 275e^{-t\left(\frac{1}{5}\ln\left(11/9\right)\right)}$

$$\frac{75}{275} = \left[e^{\ln(11/9)}\right]^{-t/5}$$ ✓

$$\frac{3}{11} = \left(\frac{11}{9}\right)^{-t/5}$$

$$\frac{3}{11} = \left(9/11\right)^{t/5}$$

$$\log\left(\frac{3}{11}\right) = \frac{t}{5}\log\left(\frac{9}{11}\right)$$

$$\frac{\log(3/11)}{\log(9/11)} = \frac{t}{5}$$

$$t = 5\frac{\log(3/11)}{\log 9/11}$$ ✓

$= 32$ minutes (to nearest minute)

or if rounded up 33 minutes (accept this also)

(32·37351077 minutes)

14 a) i) $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$ ✓

$= S$

$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$

$= S\frac{dr}{dt}$ (*) ✓

a ii) $\frac{dV}{dt} = -3S$

using (*) $S\frac{dr}{dt} = -3S$

$\frac{dr}{dt} = -3$

$r = -3t + c$

When $t = 0$, $r = 24$

$\therefore$ $24 = c$

$r = -3t + 24$ ✓

when $r = 3$,

$3 = -3t + 24$

$3t = 21$

$t = 7$ ✓

It takes 7 minutes to reduce radius to one-eighth its original radius

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14 b)

$$\text{proj}_{\underset{\sim}{v}}\,\underset{\sim}{u} = \frac{\underset{\sim}{u}\cdot\underset{\sim}{v}}{\underset{\sim}{v}\cdot\underset{\sim}{v}}\,\underset{\sim}{v}$$

$$= \frac{1\times + \times 2}{-1\times -1 - 2\times}\,(-\underset{\sim}{i} + 2\underset{\sim}{j})$$ ✓

$$= \frac{1}{5}\,(-\underset{\sim}{i} + 2\underset{\sim}{j})$$ ✓

14 (c)

$$\overrightarrow{DB} = \frac{3}{2}\,\underset{\sim}{a}$$

$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$$

$$\underset{\sim}{b} = \overrightarrow{AD} + \overrightarrow{DB} - \overrightarrow{CB}$$ ✓

$$= \underset{\sim}{a} + \frac{3}{2}\underset{\sim}{a} - \underset{\sim}{c}$$

$$= \frac{5}{2}\underset{\sim}{a} - \underset{\sim}{c}$$

$$= \frac{1}{2}\,(5\underset{\sim}{a} - 2\underset{\sim}{c})$$ ✓

14 d) 7 people may be arranged in a circle in $6!$ ways ✓

The mother and father may stand together in $5! \times 2$ ways ✓

∴ Total ways that mother and father do not stand together is $6! - 5! \times 2$

$= 480$

15 a) $\int \cos^2 \frac{x}{3} \, dx$

$= \frac{1}{2} \int \cos \frac{2x}{3} + 1 \, dx$ ✓

$= \frac{1}{2} \left( \frac{3}{2} \sin \frac{2x}{3} + x \right) + c$ ✓

b) $V = \pi \int_{\pi/4}^{\pi/3} \tan^2 y \, dy$ ✓

$\boxed{\begin{array}{l} y = \tan^{-1} x \\ \tan y = x \end{array}}$

$= \pi \int_{\pi/4}^{\pi/3} \sec^2 y - 1 \, dy$ ✓

$= \pi \left[ \tan y - y \right]_{\pi/4}^{\pi/3}$ ✓

$= \pi \left( \sqrt{3} - \pi/3 - (1 - \pi/4) \right)$

$= \pi \left( \sqrt{3} - 1 - \pi/12 \right)$ units$^3$ ✓

$\doteqdot 1.477$ (3 d.p.)

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15 c) i) $\frac{d}{dx}\left[x\sin^{-1}x + \sqrt{1-x^2}\right]$

$= \sin^{-1}x \times 1 + x \times \frac{1}{\sqrt{1-x^2}} + \frac{1}{2}(1-x^2)^{-1/2} \times -2x$ ✓

$= \sin^{-1}x + \frac{x}{\sqrt{1-x^2}} - \frac{x}{\sqrt{1-x^2}}$ ✓

$= \sin^{-1}x$

15 c ii) $\int_{1/2}^{1} \sin^{-1}x \, dx$

$= \left[x\sin^{-1}x + \sqrt{1-x^2}\right]_{1/2}^{1}$

$= 1 \times \sin^{-1}(1) + 0 - \left(\frac{1}{2}\sin^{-1}\left(\frac{1}{2}\right) + \sqrt{\frac{3}{4}}\right)$ ✓

$= \pi/2 - \left(\frac{\pi}{12} + \frac{\sqrt{3}}{2}\right)$

$= \frac{5\pi}{12} - \frac{\sqrt{3}}{2}$ ✓

$= \frac{5\pi - 6\sqrt{3}}{12}$

16 a) i) LHS $= \underset{\sim}{a} \cdot (\underset{\sim}{a} + \underset{\sim}{b})$

$= \underset{\sim}{a} \cdot \underset{\sim}{a} + \underset{\sim}{a} \cdot \underset{\sim}{b}$

$= |\underset{\sim}{a}|^2 + \underset{\sim}{a} \cdot \underset{\sim}{b}$

$= |\underset{\sim}{b}|^2 + \underset{\sim}{a} \cdot \underset{\sim}{b}$ since OACB is a rhombus ✓

$= \underset{\sim}{b} \cdot \underset{\sim}{b} + \underset{\sim}{a} \cdot \underset{\sim}{b}$

$= \underset{\sim}{b} \cdot \underset{\sim}{b} + \underset{\sim}{b} \cdot a$

$= \underset{\sim}{b} \cdot (\underset{\sim}{b} + \underset{\sim}{a})$ ✓

$= \underset{\sim}{b} \cdot (\underset{\sim}{a} + \underset{\sim}{b})$

= RHS

16 a ii) $\underset{\sim}{a} \cdot (\underset{\sim}{a} + \underset{\sim}{b}) = \underset{\sim}{b} \cdot (\underset{\sim}{a} + \underset{\sim}{b})$

$|\underset{\sim}{a}| \times |\underset{\sim}{a} + \underset{\sim}{b}| \cos \alpha = |\underset{\sim}{b}| \times |\underset{\sim}{a} + \underset{\sim}{b}| \cos \beta$ ✓

$|\underset{\sim}{a}| \times |\underset{\sim}{a} + \underset{\sim}{b}| \cos \alpha = |\underset{\sim}{a}| \times |\underset{\sim}{a} + \underset{\sim}{b}| \cos \beta$ ✓

$\cos \alpha = \cos \beta$

$\alpha = \beta$

$\therefore$ OC bisects $\angle AOB$

FORT STREET HIGH SCHOOL
WRITING PAPER

NESA Student number

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16 b i) $LHS = \cos 3\theta$

$= \cos(2\theta + \theta)$

$= \cos 2\theta \cos\theta - \sin 2\theta \sin\theta$ ✓

$= (2\cos^2\theta - 1)\cos\theta - 2\sin\theta\cos\theta\sin\theta$

$= 2\cos^3\theta - \cos\theta - 2\sin^2\theta\cos\theta$

$= 2\cos^3\theta - \cos\theta - 2\cos\theta(1-\cos^2\theta)$

$= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta$ ✓

$= 4\cos^3\theta - 3\cos\theta$

16 b ii) $8x^3 - 6x = -\sqrt{3}$ $(\div 2)$

$4x^3 - 3x = -\sqrt{3}/2$

Let $x = \cos\theta$,

$4\cos^3\theta - 3\cos\theta = -\sqrt{3}/2$

$\cos 3\theta = -\sqrt{3}/2$ ✓ for $0 \le 3\theta \le 6\pi$

$3\theta = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$= \pi - \frac{\pi}{6}, \pi + \pi/6, 3\pi - \pi/6, 3\pi + \pi/6, 5\pi - \pi/6, 5\pi + \pi/6$

$= \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{17\pi}{6}, \frac{19\pi}{6}, \frac{29\pi}{6}, \frac{31\pi}{6}$ ✓

$\theta = 5\pi/18, 7\pi/18, \frac{17\pi}{18}, \frac{19\pi}{18}, \frac{29\pi}{18}, \frac{31\pi}{18}$

for $0 \le \theta \le 2\pi$

but $x = \cos\theta$

$x = \cos\frac{5\pi}{18}, \cos\frac{7\pi}{18}, \cos 17\pi/18$ ✓

Note $\cos\frac{19\pi}{18} = \cos(\pi + \pi/18) = \cos(\pi - \pi/18) = \cos 17\pi/18$

P.T.O.

$$\cos\frac{29\pi}{18} = \cos\left(\pi + \frac{11\pi}{18}\right) = \cos\left(\pi - 11\pi/18\right) = \cos\frac{7\pi}{18}$$

$$\cos\frac{31\pi}{18} = \cos\left(\pi + \frac{13\pi}{18}\right) = \cos\left(\pi - \frac{13\pi}{18}\right) = \cos\frac{5\pi}{18}.$$